

No deconfinement in QCD ?

L. Ya. Glozman^{1,*}

¹*Institut für Physik, FB Theoretische Physik, Universitätsplatz 5, 8010 Graz, Austria*

At a critical temperature QCD in the chiral limit undergoes a chiral restoration phase transition. Above the phase transition the quark condensate vanishes. The Banks-Casher relation connects the quark condensate to a density of the near-zero modes of the Dirac operator. In the Nambu-Goldstone mode the quasi-zero modes condense around zero, $\lambda \rightarrow 0$, and provide a nonvanishing quark condensate. The chiral restoration phase transition implies that above the critical temperature there is no any longer a condensation of the Dirac modes around zero. If a $U(1)_A$ symmetry is also restored and a gap opens in the Dirac spectrum then the Euclidean QCD action with N_f flavors is $SU(2N_f) \supset SU(N_f)_L \times SU(N_f)_R \times U(1)_A$ -symmetric. This symmetry implies that a free (deconfined) propagation of quarks in Minkowski space-time is impossible. This means that the high-temperature phase of QCD is not of a quark-gluon plasma origin and has a more complicated structure.

I. INTRODUCTION

Classically QCD with N_F degenerate flavors in a finite volume V and without exact zero modes of the Dirac operator (which are irrelevant in the $V \rightarrow \infty$ limit) has a $SU(2N_f) \supset SU(N_f)_L \times SU(N_f)_R \times U(1)_A$ symmetry [1]. A symmetry of hadrons in this case is $SU(2N_F) \times SU(2N_F)$ for mesons and $SU(2N_F) \times SU(2N_F) \times SU(2N_F)$ for baryons, which is a model-independent, pure analytical statement. The axial anomaly breaks the $U(1)_A$ symmetry, and consequently also the $SU(2N_F)$ symmetry.

In the thermodynamic limit $V \rightarrow \infty$ the lowest lying eigenmodes of the Dirac operator condense around zero (the so-called near-zero modes) and provide according to the Banks-Casher relation a nonvanishing quark condensate. If effects of anomaly and of spontaneous breaking of chiral symmetry are encoded in the same near-zero modes, then a truncation of the near-zero modes should lead to a large symmetry of hadrons mentioned above. It explains naturally previous lattice observations of emergence of a symmetry of hadrons, that is larger than the chiral $SU(N_f)_L \times SU(N_f)_R \times U(1)_A$ symmetry of the QCD Lagrangian, after an artificial subtraction of the near-zero modes of the Dirac operator [2–5].

These results have a very nontrivial implication for QCD above the chiral restoration phase transition at a critical temperature T_c . Here we discuss the chiral limit, where QCD undergoes a chiral restoration phase transition [8] (at finite quark masses the phase transition converts into a fast cross-over, as it follows from lattice measurements).

In this note we address QCD at zero chemical potential, because in this case there exists a rigorous connection between the quark condensate of the vacuum and a density of modes of the Dirac operator around zero - the Banks-Casher relation. In the Nambu-Goldstone mode of chiral symmetry, i.e. below the phase transition,

the modes of the Dirac operator condense around zero, $\lambda \rightarrow 0$, and provide a nonvanishing quark condensate.

Above the phase transition the quark condensate of the vacuum vanishes. If in addition the $U(1)_A$ symmetry is also restored and a gap opens in the Dirac spectrum around $\lambda = 0$ [10, 11]¹ then the Euclidean QCD becomes $SU(2N_f)$ symmetric. Such a symmetry prohibits in Minkowski space-time a propagation of a free deconfined massless quark, because the kinetic quark term manifestly breaks the $SU(2N_f)$ symmetry. The only possibility is that the chirally symmetric quarks are confined into $SU(2N_F) \times SU(2N_F)$ and $SU(2N_F) \times SU(2N_F) \times SU(2N_F)$ symmetric "hadrons".

II. $SU(2N_F)$ HIDDEN CLASSICAL SYMMETRY OF QCD

In this section we review some results of ref. [1]. Non-perturbatively QCD is defined in Euclidean space-time in a finite box with a volume V with the lattice ultra-violet regularization. Consider the Lagrangian in Euclidean space-time for N_F degenerate quark flavors in a given gauge background,

$$\mathcal{L} = \Psi^\dagger(x)(\gamma_\mu D_\mu + m)\Psi(x) \quad (1)$$

with

$$D_\mu = \partial_\mu + ig \frac{t^a}{2} A_\mu^a, \quad (2)$$

where A_μ^a is the gluon field configuration and t^a are the $SU(3)$ -color generators. Note that in Euclidean space-time only the Hermitian conjugation, $\Psi^\dagger(x)$, can be used

¹ There is no agreement at the moment between different lattice groups on the issue of $U(1)_A$ restoration [12]. A disagreement comes from technical issues, see discussion in ref. [13]. We will assume that conclusions of the JLQCD collaboration [10, 11, 13] are correct but certainly a controversy between different lattice groups should be resolved.

* leonid.glozman@uni-graz.at

(not a Dirac adjoint bispinor) and the fields $\Psi^\dagger(x)$ and $\Psi(x)$ are independent from each other.²

The hermitian Dirac operator for a quark in a given gluonic configuration, $i\gamma_\mu D_\mu$, has in a finite volume a discrete spectrum with real eigenvalues λ_n :

$$i\gamma_\mu D_\mu \Psi_n(x) = \lambda_n \Psi_n(x). \quad (3)$$

The nonzero eigenvalues come in pairs $\pm\lambda_n$, because

$$i\gamma_\mu D_\mu \gamma_5 \Psi_n(x) = -\lambda_n \gamma_5 \Psi_n(x). \quad (4)$$

We expand the fields $\Psi(x)$ and $\Psi^\dagger(x)$ in (1) over a complete and orthonormal set $\Psi_n(x)$:

$$\Psi(x) = \sum_n c_n \Psi_n(x), \quad \Psi^\dagger(x) = \sum_k \bar{c}_k \Psi_k^\dagger(x), \quad (5)$$

where \bar{c}_k, c_n are Grassmannian numbers. Then the fermionic part of the QCD partition function takes the following form

$$Z = \int \prod_{k,n} d\bar{c}_k dc_n e^{\sum_{k,n} \int d^4x \bar{c}_k c_n (\lambda_n + im) \Psi_k^\dagger(x) \Psi_n(x)}. \quad (6)$$

The eigenmodes of the Dirac operator in a finite volume V can be separated into two classes. The nonzero eigenmodes, $\lambda_n \neq 0$, and exact zero modes, $\lambda = 0$. The exact zero modes satisfy the Dirac equation,

$$\gamma_\mu D_\mu \Psi_0(x) = 0. \quad (7)$$

They are chiral, L or R . They appear only in gauge configurations with a nonzero global topological charge. The difference of numbers of the left-handed and right-handed zero modes is fixed according to the Atiyah-Singer theorem by the global topological charge of the gauge configuration. It is well understood that these exact zero modes are completely irrelevant since their contributions to the Green functions and observables vanish in the thermodynamic limit $V \rightarrow \infty$ as $1/V$, see e.g. refs. [14–16]. Consequently, in the finite-volume calculations we can ignore (or subtract) the exact zero-modes.

Now we will analyse symmetry properties of the partition function above assuming that there are no exact zero modes. The $SU(2)_{CS} \supset U(1)_A$ transformations are defined as [6, 7]

$$\Psi \rightarrow \Psi' = e^{i\frac{\epsilon \cdot \Sigma}{2}} \Psi, \quad (8)$$

with the following generators

$$\Sigma = \{\gamma^4, i\gamma^5 \gamma^4, -\gamma^5\}, \quad (9)$$

that form an $SU(2)$ algebra

$$[\Sigma^i, \Sigma^j] = 2i\epsilon^{ijk} \Sigma^k. \quad (10)$$

The rotations (8) in an imaginary *chiral spin* space mix the right- and left-handed components of the fermion fields.

We can combine the $SU(2)_{CS}$ rotations with the flavor $SU(N_F)$ transformations into one larger group, $SU(2N_F)$. In the case of two flavors the $SU(4)$ transformations

$$\Psi \rightarrow \Psi' = e^{i\epsilon \cdot T/2} \Psi, \quad (11)$$

are defined through the following set of 15 generators:

$$\{(\tau^a \otimes \mathbb{1}_D), (\mathbb{1}_F \otimes \Sigma^i), (\tau^a \otimes \Sigma^i)\}, \quad (12)$$

where τ^a are isospin Pauli matrices. If the $(2N_F)^2 - 1$ -dimensional rotation vector ϵ is a constant for the whole 3+1-dim space, then the corresponding transformation is global, while with the space-dependent rotation $\epsilon(\mathbf{x})$ it is local.

Now we can directly read off the symmetry properties of the partition function (6). For any $SU(2)_{CS}$ and $SU(2N_F)$ transformation the Ψ_n and Ψ_m^\dagger Dirac bispinors transform as

$$\Psi_n \rightarrow U \Psi_n, \quad \Psi_m^\dagger \rightarrow (U \Psi_m)^\dagger, \quad (13)$$

where U is any unitary transformation from the groups $SU(2)_{CS}$ and $SU(2N_F)$, $U^\dagger = U^{-1}$. It is then obvious that the exponential part of the partition function is invariant with respect to global and local $SU(2)_{CS}$ and $SU(2N_F)$ transformations, because

$$(U \Psi_k(x))^\dagger U \Psi_n(x) = \Psi_k^\dagger(x) \Psi_n(x). \quad (14)$$

The exact zero modes, for which the equation (14), does not hold, have been subtracted from the partition function.³ In other words, QCD classically without the irrelevant exact zero modes has in a finite volume V both global and local $SU(2N_F)$ symmetries.

These $SU(2)_{CS}$ and $SU(2N_F)$ symmetries are hidden⁴ classical symmetries of QCD. The axial anomaly breaks

² Very often instead of Ψ^\dagger the $\bar{\Psi}$ notation is used in Euclidean space. Then it should be kept in mind that under Euclidean Lorentz transformations ($SO(4)$) $\bar{\Psi}$ transforms as Ψ^\dagger .

³ The exact zero modes break the $SU(2)_{CS}$ and $SU(2N_F)$ symmetries. Consequently these symmetries are absent at the QCD Lagrangian level, see a detailed discussion in ref. [1].

⁴ Because they are not seen at the Lagrangian level and become visible only when the irrelevant exact zero modes are subtracted.

the classical $U(1)_A$ symmetry. Since the $U(1)_A$ is a subgroup of $SU(2)_{CS}$, the axial anomaly breaks either the $SU(2)_{CS}$ and $SU(2N_F) \rightarrow SU(N_F)_L \times SU(N_F)_R$.

In the thermodynamic limit $V \rightarrow \infty$ the otherwise finite lowest eigenvalues λ condense around zero and provide according to the Banks-Casher relation at $m \rightarrow 0$ a nonvanishing quark condensate in Minkowski space [9]

$$\lim_{m \rightarrow 0} \langle 0 | \bar{\Psi}(x) \Psi(x) | 0 \rangle = -\pi \rho(0). \quad (15)$$

Here a sequence of limits is important: first an infinite volume limit is taken and only then - a chiral limit. The quark condensate in Minkowski space-time, $\langle 0 | \bar{\Psi}(x) \Psi(x) | 0 \rangle$, breaks all $U(1)_A$, $SU(N_F)_L \times SU(N_F)_R$, $SU(2)_{CS}$ and $SU(2N_F)$ symmetries to $SU(N_F)_V$. Consequently, the hidden classical $SU(2)_{CS}$ and $SU(2N_F)$ symmetries are broken both by the anomaly and dynamically.

III. RESTORATION OF $SU(2)_{CS}$ AND $SU(2N_F)$ AT HIGH TEMPERATURE

The hidden classical $SU(2)_{CS}$ and $SU(2N_F)$ symmetries are broken by the anomaly and by the quark condensate. Above the chiral restoration phase transition the quark condensates vanishes. If in addition the $U(1)_A$ symmetry is also restored [10, 11, 13] and a gap opens in the Dirac spectrum, then it follows that above the critical temperature the $SU(2)_{CS}$ and $SU(2N_F)$ symmetries are manifest.

IV. NO FREE DECONFINED QUARKS ABOVE THE CHIRAL RESTORATION PHASE TRANSITION

What do these $SU(2)_{CS}$ and $SU(2N_F)$ symmetries of QCD imply for Minkowski space-time, where we live? They imply that there cannot be deconfined free quarks.

Assume that above T_c QCD is in a deconfined phase. Then, according to the definition of deconfinement and of the quark-gluon plasma phase, there must be free propagating quarks. Free propagating quarks, presumably interacting with gluons, are solutions of the Dirac equation and have the following Lagrangian

$$\bar{\Psi} i \gamma^\mu D_\mu \Psi = \bar{\Psi} i \gamma^0 D_0 \Psi + \bar{\Psi} i \gamma^k D_k \Psi. \quad (16)$$

The first term describes an interaction of the quark charge density with the chromo-electric part of the gluonic field and the second term contains a kinetic term for a free quark as well as an interaction of the spatial current density with the chromo-magnetic field.

While the chromo-electric part of the Dirac Lagrangian is invariant under global and space-local $SU(2)_{CS}$ and $SU(2N_F)$ transformations, the kinetic term and the

quark - chromo-magnetic field interaction - are not. This necessarily implies that there are no free on-shell propagating quarks in Minkowski space-time, because the kinetic term manifestly breaks the $SU(2N_F)$ symmetry. The latter statement is equivalent to the well-known fact that for the on-shell massless fermions in Minkowski space chirality is a conserved quantum number. In this case there is no $SU(2)_{CS}$ symmetry, that mixes the left- and the right-handed fermions.

Also above T_c QCD is in a confining regime.

In contrast, color-singlet $SU(2N_F)$ -symmetric "hadrons" are not prohibited by the restored hidden $SU(2N_F)$ symmetry of QCD and can freely propagate.

V. DISCUSSION

Early arguments about deconfinement at high temperature and transition to the quark-gluon plasma are all based on perturbative derivation of a Debye screening of the color charge. QCD is however never perturbative and what these perturbative calculations mean is not clear. Such calculations are self-contradictory: They rely on unconfined quarks and gluons and at the same time try to address confining properties without any clear definition what deconfinement would mean. While something drastic might indeed happen with gluodynamics at high T , whether it means deconfinement or not is by far not clear.

The Wilson and Polyakov loop criteria of confinement-deconfinement are applicable only for a pure glue theory. While lattice measurements of the Wilson and Polyakov loops (and of related Z_3 symmetry) do show that indeed some properties of a pure glue theory rapidly change at the critical temperature, it is by far not clear whether it means deconfinement or not. To conclude about confinement-deconfinement one invokes as an intermediate step an *interpretation* of the Wilson loop and of a correlator of the Polyakov loops as a "potential" between the static color charges. What this "potential" would mean for quarks that move and whether they are confined or not is not clear.

Here in contrast we rely on the truly nonperturbative and rigorous Banks-Casher relation and on a symmetry of QCD above the chiral restoration phase transition. Namely, we have shown that given manifest $SU(N_F)_L \times SU(N_F)_R$ and $U(1)_A$ chiral symmetries the actual symmetry of QCD with N_F degenerate flavors is $SU(2N_F)$ that prohibits in Minkowski space-time an on-shell propagation of a free deconfined quark.

VI. PREDICTIONS

Appearance of the $SU(2)_{CS}$ and of $SU(2N_F)$ symmetries at $T > T_c$ can be directly tested on the lattice. By definition QCD is said to be symmetric under some symmetry group U if the diagonal correlation functions cal-

culated with a set of operators O_1, O_2, \dots that form an irreducible representation of the group U are identical, and if the off-diagonal cross-correlators vanish.

Transformation properties of meson and baryon operators under $SU(2)_{CS}$ and $SU(2N_F)$ groups are given in refs. [5, 7]. In particular, three isovector $J = 1$ mesonic operators $\bar{\Psi}\vec{\tau}\gamma^i\Psi, (1^{--})$; $\bar{\Psi}\vec{\tau}\gamma^0\gamma^i\Psi, (1^{--})$; $\bar{\Psi}\vec{\tau}\gamma^0\gamma^5\gamma^i\Psi, (1^{+-})$ form an irreducible representation of $SU(2)_{CS}$. One expects that below T_c all three diagonal correlators will be different and the off-diagonal cross-correlator of two (1^{--}) operators will be not zero. Above T_c a $SU(2)_{CS}$ restoration requires that all three diagonal correlators should become identical after a common normalization at some point and the off-diagonal correlator of two (1^{--}) currents should vanish. A restoration of

$SU(2)_{CS}$ and of $SU(N)_L \times SU(N)_R$ (the latter can be tested e.g. through a coincidence of the diagonal correlators with the vector- and axial-vector currents) implies a restoration of $SU(2N_F)$.

A similar prediction can be made with the baryon operators.

ACKNOWLEDGMENTS

The author thanks T. D. Cohen, C. Gatttringer and C. B. Lang for a careful reading of the ms. We acknowledge partial support from the Austrian Science Fund (FWF) through the grant P26627-N27.

-
- [1] L. Y. Glozman, arXiv:1511.05857 [hep-ph].
 - [2] M. Denissenya, L. Y. Glozman and C. B. Lang, Phys. Rev. D **89**, 077502 (2014).
 - [3] M. Denissenya, L. Y. Glozman and C. B. Lang, Phys. Rev. D **91**, 034505 (2015).
 - [4] M. Denissenya, L. Y. Glozman and M. Pak, Phys. Rev. D **91**, 114512 (2015).
 - [5] M. Denissenya, L. Y. Glozman and M. Pak, Phys. Rev. D **92**, no. 7, 074508 (2015).
 - [6] L. Y. Glozman, Eur. Phys. J. A **51**, 27 (2015).
 - [7] L. Y. Glozman and M. Pak, Phys. Rev. D **92**, 016001 (2015).
 - [8] R. D. Pisarski and F. Wilczek, Phys. Rev. D **29**, 338 (1984).
 - [9] T. Banks and A. Casher, Nucl. Phys. B **169**, 103 (1980).
 - [10] G. Cossu, S. Aoki, H. Fukaya, S. Hashimoto, T. Kaneko, H. Matsufuru and J. I. Noaki, Phys. Rev. D **87**, no. 11, 114514 (2013) [Phys. Rev. D **88**, no. 1, 019901 (2013)].
 - [11] A. Tomiya, G. Cossu, H. Fukaya, S. Hashimoto and J. Noaki, PoS LATTICE **2014**, 211 (2015) [arXiv:1412.7306 [hep-lat]]; G. Cossu *et al.* [JLQCD Collaboration], arXiv:1511.05691 [hep-lat];
 - [12] S. Sharma, V. Dick, F. Karsch, E. Laermann and S. Mukherjee, arXiv:1602.02197 [hep-lat].
 - [13] S. Aoki [JLQCD Collaboration], PoS(CD15) (2015) 045 [arXiv:1603.00997 [hep-lat]].
 - [14] H. Leutwyler and A. V. Smilga, Phys. Rev. D **46**, 5607 (1992).
 - [15] R. Brower, S. Chandrasekharan, J. W. Negele and U. J. Wiese, Phys. Lett. B **560**, 64 (2003).
 - [16] S. Aoki, H. Fukaya and Y. Taniguchi, Phys. Rev. D **86**, 114512 (2012).